To localize the robot, we can accumulate a change of position from the current frame vs. the previous. Then sum the changes.

Assuming the robot is constantly going through “curved” motion, Approximate robot trajectory as Arc segments of a large circle would be appropriate.

Given:
Δh Recorded change by horizontal encoder. Positive to Left.
Δv Recorded change by Vertical encoder. Positive to Forward.
θ Current heading of robot reported by IMU, positive to counter-clockwise.
θ_p Heading of the previous frame.

To find the radius of motion, we derive the following:

$$\Delta h = (R_h + r_h) \cdot (\theta - \theta_p)$$

$$\Delta v = (R_v + r_v) \cdot (\theta - \theta_p)$$

The radius of motion on tangent to horizontal encoder

$$R_h = \frac{\Delta h}{\theta - \theta_p} - r_h$$

The radius of motion on tangent to vertical encoder

$$R_v = \frac{\Delta v}{\theta - \theta_p} - r_v$$

Given the radius of motion and heading of the robot. We know robot’s current location from the origin of rotations given:

$$\overline{r_1} = R_h \cdot [cos\,(\theta - \pi)\,\hat{x} + sin\,(\theta - \pi)\,\hat{y}] + R_v \cdot [cos\,(\theta - \pi/2)\,\hat{x} + sin\,(\theta - \pi/2)\,\hat{y}]$$

$$\overline{r_1} = R_h \cdot [-\,cos\,(\theta)\,\hat{x} - sin\,(\theta)\,\hat{y}] + R_v \cdot [sin\,(\theta)\,\hat{x} - cos(\theta)\,\hat{y}]$$

$$\overline{r_0} = R_h \cdot [cos\,(\theta_p - \pi)\,\hat{x} + sin\,(\theta_p - \pi)\,\hat{y}] + R_v \cdot [cos\,(\theta_p - \pi/2)\,\hat{x} + sin\,(\theta_p - \pi/2)\,\hat{y}]$$

$$\overline{r_0} = R_h \cdot [-\,cos\,(\theta_p)\,\hat{x} - sin\,(\theta_p)\,\hat{y}] + R_v \cdot [sin\,(\theta_p)\,\hat{x} - cos(\theta_p)\,\hat{y}]$$

$$\overline{\Delta r} = \overline{r_1} - \overline{r_0}$$

We have the following

$$\Delta x = R_h \cdot [-\,cos\,(\theta) + cos\,(\theta_p)] + R_v \cdot [\,sin\,(\theta) - sin\,(\theta_p)]$$

$$\Delta y = R_h \cdot [-\,sin\,(\theta) + sin\,(\theta_p)] + R_v \cdot [\,-\,cos(\theta) + cos(\theta_p)]$$

$$\bar{r} = \overline{\Delta r} + \overline{r_p}$$